Chapter:→3:

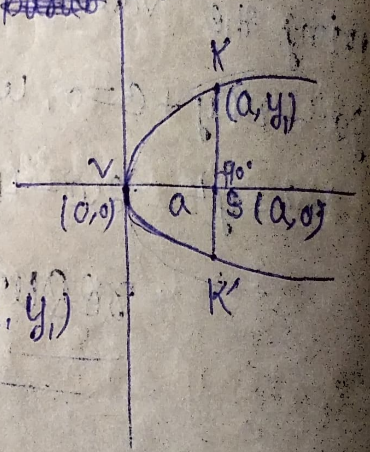
Conic-Section

① QN.→ Definition of Latus rectum, and find the latus-rectum of $y^2 = 4ax$.

Ans.→ Latus-rectum :→ The double ordinate KSK' passing through the focus and ⊥ to the axis of the parabola is know as latus rectum of the parabola.

~~Let $y^2 = 4ax$ be the eqn. of parabola~~

Let $y^2 = 4ax$ be the eqn. of ~~parab~~ parabola



Hence, the focus S(a, 0)

and, $VS = a$

Hence the Conclude of K is (a, y_1)

$\therefore KS = y_1$

As the point (a, y_1) lies on the Parabola

$$y^2 = 4ax$$

$$or, y_1^2 = 4aa$$

$$y_1^2 = 4a^2$$

$$\therefore KS = 2a \quad y_1 = 2a$$

$$KSK' = 4a$$

$\therefore$ latus-rectum of parabola $= 4a$

② QN.→ Definition of Parabola.

Ans.→ Parabola :→ A Parabola is the locus of a point which moves such that the distance from a fixed point (focus) is equal to the distance from the fixed line

(directrix)

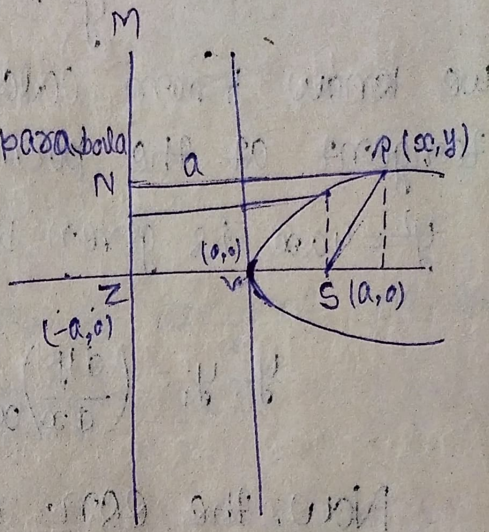
③ QN:→ To find the eqn. of the standard ~~para~~ parabola.

Ans.→ Let,

S = focus

V = vertex

MZ = directrix of the parabola



Now, we want to know the eqn. of parabola.

from S, draw SZ ⊥ MZ

Let V = mid point of SZ

$\therefore VS = ZV$

Take V as origin i.e. V(0,0) and ZS = 2a,

i.e. VS = a, ZV = a

Hence, Co-ordinate of S(a, 0)

and ——————— Z(−a, 0)

Let P(x, y) be any point on the parabola

Hence from the defⁿ of parabola

$PS = PN$

$PN = (a + x)$

$PS^2 = PN^2$

or, $(x-a)^2 + (y-0)^2 = (a+x)^2$

or, $\cancel{x^2} - \cancel{a^2} - 2ax + y^2 = \cancel{a^2} + 2ax + \cancel{x^2}$

$\therefore y^2 = 2ax + 2ax = 4ax$

required equation of Parabola

(4) Qn. → To find the equation of the tangent at the point (x_1, y_1) on the parabola $y^2=4ax$

Ans. → ∵ the eqn. of parabola is

$$y^2 = 4ax \quad \text{—} (1)$$

we know from calculus that the eqn. of tangent at the point (x_1, y_1) to the curve $y^2=4ax$, is given by

$$y - y_1 = \left(\frac{dy}{dx}\right)_{x_1, y_1} (x - x_1) \quad \text{—} (2)$$

Now, the eqn. of parabola

$$y^2 = 4ax$$

D. b. s. w. r. t. x, we have

$$\cancel{2}y\frac{dy}{dx} = \cancel{4}^{2}a$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

$$\therefore \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{2a}{y_1}$$

Now, Putting the values of $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$ the eqn. (2), we have,

$$y - y_1 = \frac{2a}{y_1}(x - x_1)$$

or, $yy_1 - y_1^2 = 2ax - 2ax_1$

$yy_1 = y_1^2 + 2ax - 2ax_1 + 2ax_1 - 2ax_1$

or, $yy_1 = y_1^2 - 4ax_1 + 2ax + 2ax_1$

$\because (x_1, y_1)$ lies on the ~~curve~~ ①.

$y_1^2 = 4ax_1$

or, $y_1^2 - 4ax_1 = 0$

$\therefore yy_1 = 2a(x + x_1)$

required eqn. of tangent at the pt. (x_1, y_1) to the parabola $y^2 = 4ax$

⑤ Qn. → Prove that through any point three normals can be drawn to the parabola

~~or, To find the equation of the normal at the point (x_1, y_1) of the parabola $y^2 = 4ax$.~~

Ans. → Eqn. of tangent to the parabola $y^2 = 4ax$ at the pt. (x_1, y_1)

$$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$$

m of the line $m_1 = \frac{dy}{dx}$

Normal is a st. line passing through the pt. of contact and $\perp$ to the tangent at the pt. of contact

Let $m_2 =$ slope gradient of normal

$\therefore m_1 \times m_2 = -1$

$\frac{dy}{dx} \cdot m_2 = -1$

$m_2 = -\frac{dx}{dy}$

Hence the eqn. of normal at the pt. (x_1, y_1) to the Parabola

$$y - y_1 = -\left(\frac{dx}{dy}\right)(x - x_1)$$

required eqn. of Normal.

⑥ Q No.→ To find the equation of the normal at the point (x_1, y_1) of the parabola $y^2 = 4ax$

Ans.→ We know that the eqn. of tangent at the point (x_1, y_1) to the parabola $y^2 = 4ax$ is

$$yy_1 = 2a(x + x_1)$$

$$\text{or, } 2ax - yy_1 + 2ax_1 = 0 \quad -(1)$$

$\therefore$ m of this tangent $m_1 = \cancel{-}\frac{2a}{\cancel{-}y_1} = \frac{2a}{y_1}$

$\because$ the eqn. of a line passing through (x_1, y_1) is given by

$$y - y_1 = m(x - x_1) \quad -(2)$$

If line (2) is $\perp$ to (1) at the point (x_1, y_1)

then $\frac{2a}{y_1} \times m = -1$

$$m = -\frac{y_1}{2a}$$

Hence the eqn. of normal

$$y - y_1 = m(x - x_1)$$

or, $y - y_1 = -\frac{y_1}{2a}(x - x_1)$